Indian Statistical Institute, Bangalore

B. Math.

First Year, Second Semester

Linear Algebra II

Back paper Examination — Instructor: B V Rajarama Bhat

Total Marks 7x15=105 — Date : June 3, 2026

Maximum marks: 100 — Time: 3 hours

Here $M_n(\mathbb{C})$ denotes the vector space of $n \times n$ complex matrices. The standard inner product is considered unless specified otherwise.

(1) Let $B \in M_n(\mathbb{C})$. Show that the determinant B is same as the determinant of the transpose of B. [15]

(2) Suppose C is a 2×3 matrix. Show that the determinant of C^*C is zero, but the determinant of CC^* may not be zero. [15]

(3) Let $A \in M_n(\mathbb{C})$. Suppose $a_1, a_2, \ldots, a_k$ are some distinct eigenvalues of A, with respective eigenvectors $v_1, v_2, \ldots, v_k$:

$$v_j \neq 0,\ Av_j = a_j v_j,\ 1 \leq j \leq k.$$

Show that $v_1, v_2, \ldots, v_k$ are linearly independent. [15]

(4) Show that an upper triangular matrix is normal if and only if it is diagonal. [15]

(5) Obtain the spectral decomposition in the form $\sum_j a_j P_j$, where P_j's are mutually orthogonal matrices for the following matrices:

$$C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix},\ D = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

[15]

(6) Let A be a normal matrix. Show that a matrix B commutes with A if and only if it commutes with A^*. [15]

(7) Show that if a matrix M is invertible then M^{-1} is a polynomial in M and M is a polynomial in M^{-1}. [15]